



K23P 0498

Reg. No. :

Name :

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C 06: Advanced Abstract Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Find $[Q(\sqrt{2}, \sqrt{3}) : Q]$.
2. Find the primitive 5^{th} root of unity in Z_{11} .
3. Distinguish between primes and irreducibles of an integral domain.
4. Is $Z[i]$ is an integral domain ?
5. What is the order of $G(Q(\sqrt[3]{2})/Q)$?
6. Show that $\sqrt{1+\sqrt{5}}$ is algebraic over Q .

PART – B

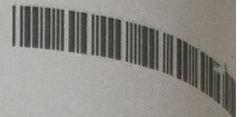
Answer **4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

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|---|---|
| 7. a) Prove that every PID is a UFD. | 7 |
| b) Prove that $Z[\sqrt{-5}]$ is an integral domain but not a UFD. | 9 |
| 8. a) State and prove Kronecker's theorem. | 8 |
| b) How could we construct a field of 4 elements ? | 8 |

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9. a) State and prove Gauss's Lemma.
b) An ideal $\langle p \rangle$ in a PID is maximal if and only if p is irreducible.
c) Prove that every Euclidian domain is PID.

Unit – II

10. a) If α and β are constructible real numbers, then $\alpha + \beta$, $\alpha - \beta$, $\alpha\beta$ and α/β , if $\beta \neq 0$.
b) If E is a finite of characteristic P , then E contains exactly P^n elements for some positive n .
11. a) Prove that trisecting an angle is impossible.
b) Prove that a finite field $GF(P^n)$ of P^n elements exists for every prime power P^n .
12. a) State and prove Conjugation isomorphism theorem.
b) Define Frobenius automorphism. Also prove that $F_{\{\sigma_p\}} \cong Z_p$.

Unit – III

13. a) A Field E , where $F \leq E \leq K$, is a splitting field over F if and only if every automorphism of $\bar{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of F leaving F fixed.
b) Let $f(x)$ be irreducible in $F[x]$. Then prove that all zeros of $f(x)$ in $\bar{F}$ have the same multiplicity.
14. a) Prove that every finite field is perfect.
b) Find the splitting field of $x^3 - 2$ over Q .
15. a) State the main theorem of Galois Theory.
b) State and prove Primitive Element theorem.